Application of real gas models to the problem of high-speed flow around a body

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The paper presents results of computational simulation of the high-speed airflow around an axisymmetric body using various models that include ideal gas described by the Mendeleev—Clapeyron equation, real gas described by the Redlich—Kwong equation and user model approximating the empirical data. The user model is characterized by its own program code for the two-parameter approximation of the air thermophysical properties and taking into account in this context changes accompanying dissociation phenomena occurring at high temperatures without simulating physical and chemical transformations in the multicomponent gas mixture. The purpose of this study is to evaluate differences in the gas-dynamic flow pattern, shock-wave structure and thermal loading of the streamlined body depending on selection of the medium model. The results obtained make it possible to conclude on the need to introduce the real gas user models to reduce the error of computational simulation and ensure correct estimation of the heat flows.

Keywords: *high-speed flow, real gas, high-temperature air, computational simulation*

Introduction. At present, the development of high-speed aircraft is one of the most urgent directions in the development of aviation [1, 2]. This problem cannot be solved without large-scale research of a fundamental and applied nature [2]. A particularly important factor in research is the correctness of the design of computational experiments, since it is very difficult to conduct physical experiments and, to an even greater extent, flight tests of supersonic aircraft.

According to [2], at present, numerous groups of high-speed systems are being developed and tested. Based on the analysis of the data presented in [2], it can be assumed that the most popular is the modeling of the flow of atmospheric air around bodies with velocities corresponding to the Mach numbers M = 3...10 at an altitude of 10...50 km.

In mathematical and computational modeling for atmospheric air, the most widespread is the Mendeleev — Clapeyron equation, which is often called the generalized gas law, which describes the state of a thermally perfect gas [3]. In the case when the use of “ideal gas”, a priori will lead to large errors in the results of calculations, apply the model of “real gas” Redlich — Kwong or its modification [5].

A more accurate result of calculations in the case of a high-speed air flow around bodies would make it possible to obtain specific models of gaseous media, taking into account the change in the composition of the gas with an increase in its temperature due to the dissociation and ionization of molecules. Such models include the Kraiko model [4], the implementation of which in software packages for thermogasdynamic calculations is difficult. An easier way to create a custom model of high-temperature atmospheric air is to set interpolation polynomials to describe the basic thermophysical properties of a gas (density, heat capacity, thermal conductivity, viscosity depending on temperature and pressure) according to known empirically obtained values [5].

Formulation of the problem. Let us consider the flow of atmospheric air around an axisymmetric body at an altitude of 30 km [6]. Figure 1 shows a geometric model of the problem under consideration.

The simulation is carried out by means of the ANSYS Fluent computer software complex in a viscous stationary setting; the SST turbulence model is used as the closing relations for the RANS equations [7]. The flow velocities corresponding to the Mach numbers M = 5 are set as the boundary conditions; M = 6; 7; 8; 9 at pressure and temperature at an altitude of 30 km [6].

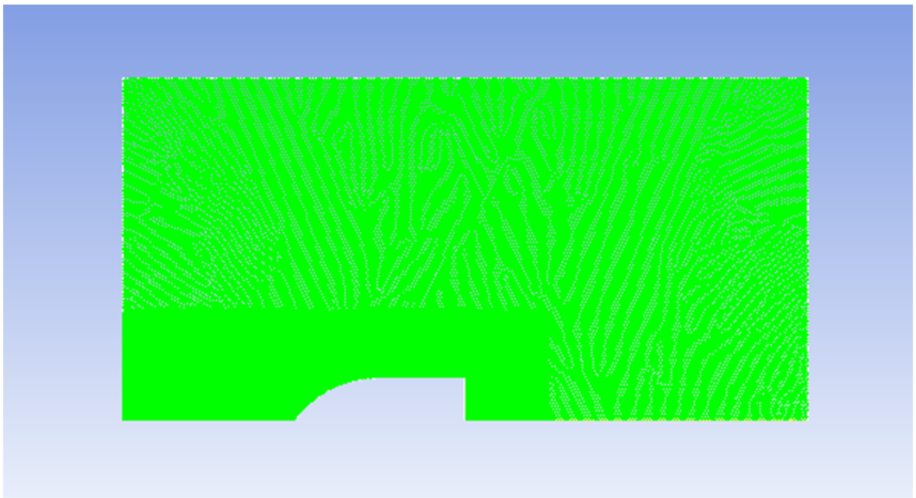


Figure 1. Geometric model of an axisymmetric body.

The following models of the environment were used as a working medium (air):

– standard model of “ideal gas” described by the Mendeleev — Clapeyron equation;

– standard model of “real gas”, described by the Redlich — Kwong equation;

– custom high temperature air model approximating empirical data [5].

The Redlich — Kwong two-parameter equation of state is described by the formula

$$P = \frac{RT}{V-b} - \frac{a}{T^{0.5}V\left(V+b\right)}, \tag{1}$$

where $a = \dfrac{Q_a R^2 T_c^{2.5}}{P_c}$, $Q_a = 0.4274802327\ldots$; $b = \dfrac{Q_b R T_c}{P_c}$, $Q_b =$ $= 0.086640350\ldots$

Equation (1) is cubic with respect to volume or compressibility factor and can also be written as

$$Z^3 - Z^2 + \left(A - B^2 - B\right)Z - AB = 0,$$

where Z is compressibility factor; $A = \dfrac{Q_a P_r}{T_r^{2.5}} = \dfrac{aP}{R^2 T^{2.5}}$; $B = \dfrac{Q_b P_r}{T_r} = \dfrac{bP}{RT}$.

The user-defined model of high-temperature air is based on the approximation of the thermophysical properties of air specified in a table [5] using separate function files (the so-called udf — user defined functions) written in C for ANSYS Fluent.

When creating a two-parameter dependence of thermophysical parameters on division and temperature based on tabular specified values, the least squares method is used. Consider the construction of an approximating plane, that is, a linear change in the considered parameter from pressure and temperature.

The approximating polynomial F for the thermophysical parameter Z (density, heat capacity, thermal conductivity or viscosity) will have the form

$$F_{i,j}\left(T_i, p_j\right) = a_1 T_i + a_2 \mathrm{p_j} + a_3,$$

where a_1, a_2, a_3 are unknown polynomial coefficients; T, p are temperature and pressure; indices i and j correspond to the number of temperature and pressure values in the approximate tables of parameters.

The standard deviation function has the form

$$\Phi = \sum_{i=1, j=1}^{n,m} \left(a_1 T_i^{\cdot} + a_2 p_j^{\cdot} + a_3 - Z_{i,j}\right)^2.$$

The system of equations for determining the coefficients of the approximating plane is compiled by equating to zero partial derivatives with respect to unknown coefficients,

$$\frac{\partial \Phi}{\partial a_1} = 2 \sum_{i=1, j=1}^{n,m} \left(a_1 T_i^{\cdot} + a_2 p_j^{\cdot} + a_3 - Z_{i,j}\right) T_i = 0;$$

$$\frac{\partial \Phi}{\partial a_2} = 2 \sum_{i=1, j=1}^{n,m} \left(a_1 T_i^{\cdot} + a_2 p_j^{\cdot} + a_3 - Z_{i,j}\right) p_j = 0; \qquad (2)$$

$$\frac{\partial \Phi}{\partial a_3} = 2\sum_{i=1,j=1}^{n,m}\left(a_1 T'_i + a_2 p'_j + a_3 - Z_{i,j}\right) = 0.$$

For further solution, it is more convenient to rewrite the system (2) in matrix form:

$$\mathrm{A} = \begin{pmatrix} \sum\limits_{i=1,j=1}^{n,m} T_i^2 & \sum\limits_{i=1,j=1}^{n,m} T'_i p'_j & \sum\limits_{i=1,j=1}^{n,m} T'_i \\ \sum\limits_{i=1,j=1}^{n,m} T'_i p'_j & \sum\limits_{i=1,j=1}^{n,m} p_j^2 & \sum\limits_{i=1,j=1}^{n,m} p'_j \\ \sum\limits_{i=1,j=1}^{n,m} T'_i & \sum\limits_{i=1,j=1}^{n,m} p'_j & n,m \end{pmatrix};$$

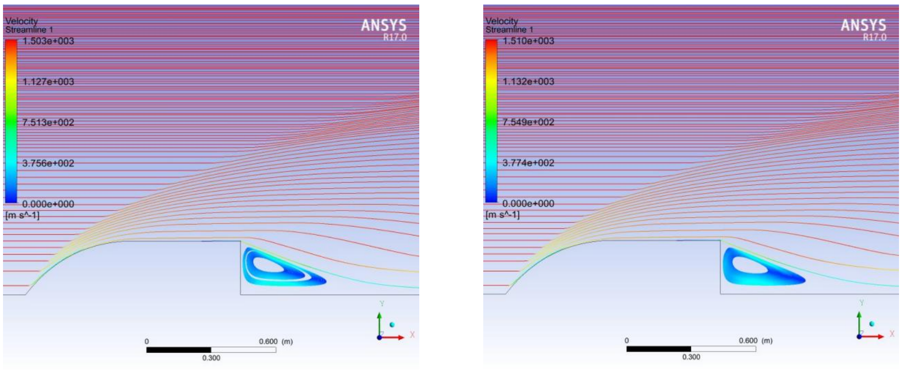
$$\mathrm{X} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}; \quad \mathrm{B} = \begin{pmatrix} \sum\limits_{i=1,j=1}^{n,m} T'_i Z_{i,j} \\ \sum\limits_{i=1,j=1}^{n,m} p'_j Z_{i,j} \\ \sum\limits_{i=1,j=1}^{n,m} Z_{i,j} \end{pmatrix}.$$

The solution of the system is carried out for thermophysical parameters, i.e. density, heat capacity, thermal conductivity or viscosity of the gas.

Thus, the change in the thermophysical properties of air is simulated without solving the physicochemical problem of calculating a multicomponent gaseous medium [8–10], which greatly simplifies calculations, reduces the time of computational simulation, and reduces the requirements for the computer technology used.

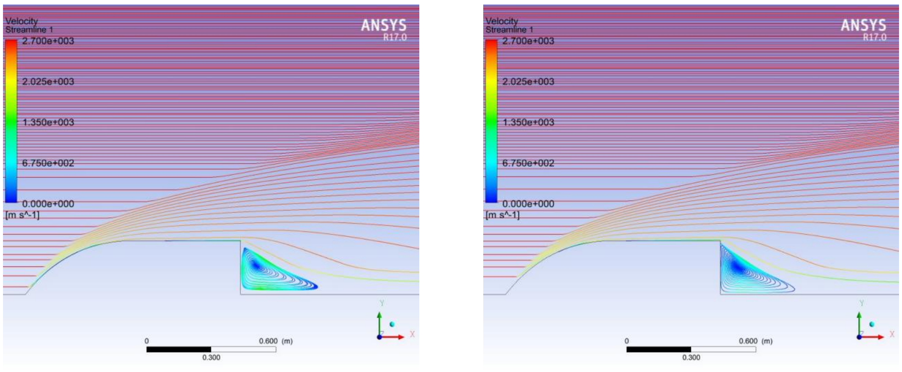
The aim of this study is to assess the differences in the gas-dynamic flow pattern, shock-wave structure, and thermal loading of a streamlined body, depending on the choice of the medium model.

Results of the study. Figures 2, 3 show patterns of streamlines colored in accordance with the flow velocity. Figures 4, 5 show the gas temperature distribution fields. The table shows the values of the stagnation temperature in the frontal part of the streamlined body, as well as the differences in the values obtained when using the standard thermally perfect gas model ("ideal gas") and a user real gas model.



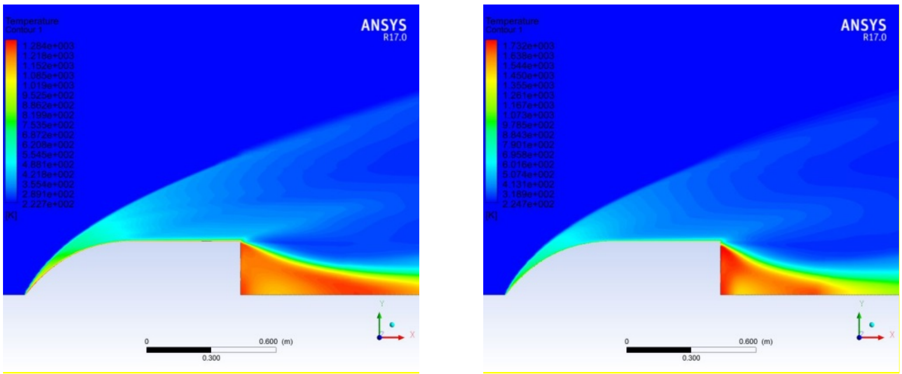
(a) (b)

Figure 2. Streamlines corresponding to flow velocities of ideal (a) and user (b) gas, M = 5.



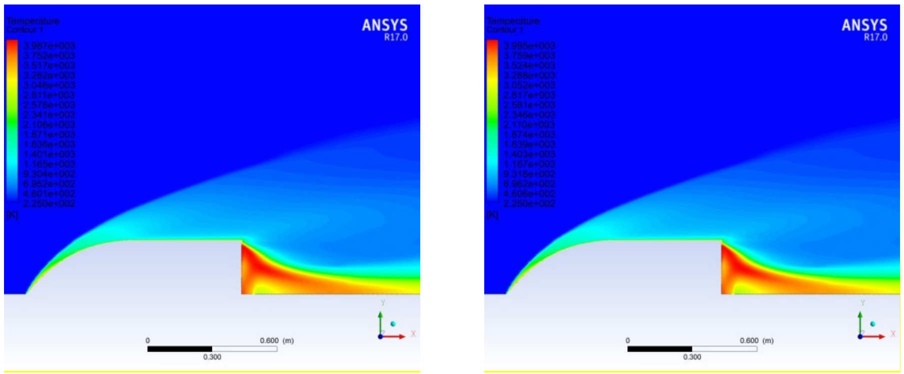
(a) (b)

Figure 3. Streamlines corresponding to flow velocities of ideal (a) and user (b) gas, M = 9.



(a) (b)

Figure 4. Temperature distribution fields of ideal (a) and user (b) gas, M = 5.



(a) (b)

Figure 5. Temperature distribution fields of ideal (a) and user (b) gas, M = 9.

Values of stagnation temperatures.

Mach number M	Gas temperature, K		$\Delta = T_{\text{ideal}} - T_{\text{user}}$, K
	T_{ideal}	T_{user}	
5	1284	1293	9
6	2286	1752	534
7	2618	2292	326
8	3337	2662	675
9	3987	2866	1121

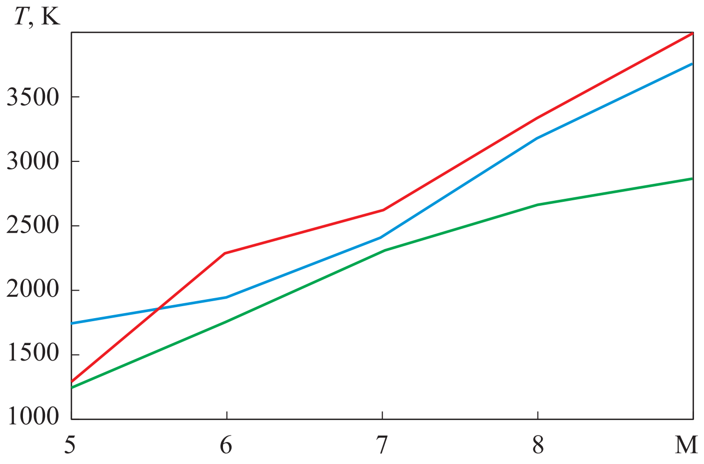


Figure 6. Dependence of the stagnation temperature T, K on the Mach number M:
🟥 — ideal gas; 🟦 — real gas; 🟩 — user gas

For a more visual display of the difference in the obtained values of the stagnation temperature, Figure 6 shows the graphs of the stagnation temperature depending on the Mach number of the incoming flow for all three considered air models.

Conclusion. The computational experiments carried out have shown that the nature of the flow and the shock-wave structure change little when varying the models of the medium. In this case, the temperature values in the vicinity of the stagnation point and on the surface of the streamlined body critically depend on the choice of the medium model. The difference between the results obtained for the stagnation temperature in the vicinity of the streamlined body when using standard medium models and a custom air model increases with an increase in the incoming flow velocity (Mach number). At supersonic Mach numbers $M \leq 5$, the standard "ideal gas" model shows the coincidence of the braking temperature with an error of no more than 5% compared to the real gas. In this case, the Redlich — Kwong gas model turns out to be inapplicable, since the error in calculating the braking temperature is greater than 25%.

With Mach numbers $5 \leq M \leq 6$, the Redlich — Kwong gas model turns out to be more efficient than the "ideal gas" model, and the error in calculating the stagnation temperature is 5...10%.

With a further increase in the flow Mach number, a discrepancy is observed in the character of the stagnation temperature change obtained using the standard and custom models. Despite the fact that the Redlich — Kwong model better describes the behavior of a real gas, its application leads to an error in calculating the stagnation temperature of 15% at the Mach number $M = 8$ and with an increase in the Mach number, the error increases nonlinearly.

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