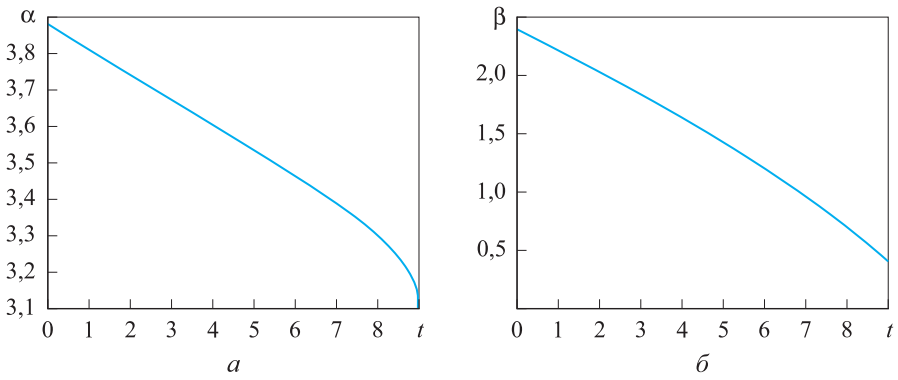

Рис. 5. Закон изменения α и β от времени t:
$а$ — $\alpha(t)$; $б$ — $\beta(t)$

Из рис. 3 и 4 видно, что расстояние в конечный момент времени между игроками, в случае реализации первым игроком метода погони, больше, и эта разница увеличивается с возрастанием величины фиксированного времени процесса.

Заключение. Авторами настоящей статьи исследована нелинейная задача встречи движущихся объектов с фиксированной стратегией наведения у одного игрока и оптимальной — у другого. С помощью принципа максимума Понтрягина задача оптимального управления сведена к краевой задаче для исходных переменных. Проведен качественный анализ системы, исследованы характерные свойства траекторий, проиллюстрированные численным моделированием.

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On a nonlinear problem of optimal rendezvous

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The paper analyzes a nonlinear problem of optimal rendezvous of two material points in the horizontal plane. The velocity of both participants is constant modulo. The aim of control is to minimize the final distance between participants under given initial conditions. The approach time is fixed. The angle between the line of sight and the velocity vector of the Participant 1 (P1) is used as a control variable. The Participant (P2) uses the proportional-navigation law. This task may be relevant when planning the approach paths of a tanker aircraft to an unmanned aerial vehicle, or in the case of intercepting an attacking unmanned aerial vehicle by a target simulator missile launched from a real target. The principle of maximum procedure allows reducing optimal control problem to the problem of analyzing the phase portrait of a system of two nonlinear differential equations. A qualitative analysis of the system is performed, the characteristic properties of the trajectories of the participants in the horizontal plane are investigated and the results of numerical solution of the boundary value problem are presented.

Keywords: *Pontryagin maximum principle, proportional-navigation law, phase portrait, optimal control problem*

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