

## Using the Bubnov—Galerkin method to calculate the nonlinear oscillations of the variable-length mathematical pendulum at final movement from one state of rest into another

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*To assess the efficiency of the Bubnov — Galerkin method and its precision compared to the numerical technique, we consider the nonlinear oscillations of the variable-length mathematical pendulum at the moving suspension. The authors have set the following task: to move the pendulum attachment point for defined time at given distance with the simultaneous suppression of natural modes at the moment of stopping. The control function with the kinematic control of final movement is specified as a series of sines. The unknown required function representing the pendulum rotational angle in accordance with the Bubnov — Galerkin method is written as the resolution with the unknown coefficients by given time functions. The work forms a system of linear algebraic equations, the order of which depends on the number of approximating functions. We consider the calculation examples at various initial settings of the problem.*

**Keywords:** *mathematical pendulum, final movement, suppression of oscillations, Bubnov — Galerkin method, nonlinear oscillations, a system of linear algebraic equations, approximating functions*

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