Non-linear vibrations in mechanical systems with one or two degrees of freedom

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The article investigates free and forced non-linear vibrations in three mechanical systems with one or two degrees of freedom. We designed our mathematical simulation of motion by reducing Lagrange equations of the second kind to their Cauchy form and subsequently integrating them numerically using a third-order Runge—Kutta method, taking into account the fact that the forces affecting the system do not depend on second derivatives of generalized coordinates. We considered those systems the typical components of which are a slider-crank mechanism, a crosshead, a physical pendulum, a spring, a damper and a planetary drive. We determined the number of equilibrium positions and their stability type for a vibration system featuring a planetary drive. We used the phase portrait and frequency response of the system to study its dynamics. Estimating the eigenfrequencies of the system via linearized models makes plotting the frequency response noticeably easier.

Keywords: *Lagrange equations, nonlinear oscillations, mathematical simulation, Runge-Kutta method, phase portrait, mechanical system, frequency response*

REFERENCES

[1] Lyapunov A.M. *Izbrannye trudy. Raboty po teorii ustoychivosti* [Selected works. Publications on stability theory]. Moscow, Nauka Publ., 2007, 576 p.

[2] Bogolyubov N.N., Mitropolskiy Yu.A. *Asimptoticheskie metody v teorii nelineynykh kolebaniy* [Asymptotic methods in theory of non-linear oscillations]. Moscow, Nauka Publ., 1974, 504 p.

[3] Krylov N.M., Bogolyubov N.N. *Vvedenie v nelineynuyu mekhaniku* [Introduction to nonlinear mechanics]. Kiev, Academy of Sciences of the UkrSSR Publ., 1937, 366 p.

[4] Gilchrist O. *Int. J. Mech. Sci*., 1961, vol. 3, pp. 286–311. DOI: 10.1016/0020-7403(61)90027-3

[5] Guskov A.M., Panovko G.Ya. *Inzhenernyy zhurnal: nauka i innovatsii — Engineering Journal: Science and Innovation*, 2012, no. 6 (6). DOI: 10.18698/2308-6033-2012-6-265

[6] Kholostova O.V. *J. Appl. Maths Mechs*, 1995, vol. 59, no. 4, pp. 553–561. DOI: 10.1016/0021-8928(95)00064-X

[7] Nicolas M. *Int. J. Non-Linear Mechanics*, 2017, no. 93, pp. 21–29. DOI: 10.1016/j.ijnonlinmec.2017.04.017

[8] Van Dooren R. *Int. J. Non-Linear Mechanics*, 1973, vol. 8, pp. 575–583. DOI: 10.1016/0020-7462(73)90007-3

[9] Volkova V.E. *Vestnik Dnepropetrovskogo natsionalnogo universiteta zheleznodorozhnogo transporta imeni akademika V. Lazaryana — Bulletin of Dnipropetrovsk National University of Railway Transport named after Academician V. Lazaryan*, 2004, no. 3, pp. 115–120.

[10] Lenci S., Mazzilli C.E. *Int. J. Non-Linear Mechanics*, 2017, vol. 94, pp. 223–234. DOI: 10.1016/j.ijnonlinmec.2017.02.017

[11] Teufel A., Steindl A., Troger H. *Communications in Nonlinear Science and Numerical Simulation*, 2006, no. 11, pp. 577–594. DOI: 10.1016/j.cnsns.2005.01.004

[12] Akulenko L.D., Nesterov S.V. *Prikladnaya matematika i mekhanika — Journal of Applied Mathematics and Mechanics*, 2013, vol. 77, no. 2, pp. 209–220. DOI: 10.1016/j.jappmathmech.2013.07.004

[13] Tkhai V.N. *Prikladnaya matematika i mekhanika — Journal of Applied Mathematics and Mechanics*, 2011, vol. 75, no. 3, pp. 430–434. DOI: 10.1016/j.jappmathmech.2011.07.007

[14] Dunin M.S., Semenov M.V., Yakuta A.A. *Fizicheskoe Obrazovanie v vuzakh — Physics in Higher Education*, 1999, vol. 5, no. 4, pp. 160–173.

[15] Ilin M.M., Kolesnikov K.S., Saratov Yu.S. *Teoriya kolebaniy* [Vibration theory]. Moscow, Bauman Moscow State Technical University, 2003, 272 p.

[16] Kolesnikov K.S., ed. *Kurs teoreticheskoy mekhaniki* [Course in theoretical mechanics]. 3rd ed. Moscow, BMSTU Publ., 2005, pp. 382–386; p. 493–554.

[17] Golubev Yu.F. *Osnovy teoreticheskoy mekhaniki* [Foundations of theoretical mechanics]. Moscow, Moscow State University Publ., 2000, pp. 397–404; pp. 539–622.

[18] Runge C.D. *Mathematische Annalen*, June 1895, vol. 46, no. 2, pp. 167–178. DOI: 10.1007/BF01446807

[19] Butenin N.V., Lunts Ya.L., Merkin D.R. *Kurs teoreticheskoy mekhaniki* [A course in theoretical mechanics]. Saint Petersburg, Lan Publ., 2009, 736 p.

[20] Berns V.A., Lysenko E.A., Dolgopolov A.V., Zhukov E.P. *Izvestiya Samarskogo nauchnogo tsentra Rossiyskoy akademii nauk — Proceedings of the Samara Scientific Center of the Russian Academy of Sciences*, 2016, vol. 18, no. 4–1, pp. 86–96.

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