Optimal thrust control for brachistochrone with viscous friction

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The study focuses on the problem of optimization of the aircraft controlled descent in homogeneous gravity field in the presence of a resisting medium and an accelerating force when moving vertically. As a means of control, lifting force and thrust force are applied. The model under consideration also describes the motion of a point on a curve, where the control variables are the normal component of the reaction force and the accelerating force. The control goal is to maximize the horizontal distance (terminal term) and minimize energy consumption (integral term) over a fixed time interval. The range maximization problem is interrelated with the Brachistochrone problem — the problem of choosing the shape of the trajectory connecting two given points in the vertical plane, the time along which will be minimal. To solve the problem, the Pontryagin maximum principle and the methods of qualitative investigation of dynamical systems are applied. Findings of the research show that extreme trajectories correspond to the motion with singular control for normal reaction of the reference curve and with regular control for thrust. Extreme control is maintained in the form of feedback on the phase variables of the initial system. The characteristic properties of the trajectories were found, which allowed us to substantiate the results obtained by other authors by means of numerical simulation, or formulated as hypotheses. Results suggest that at large time intervals extreme trajectory consists of three parts, which are output in the neighborhood of asymptotic lines, movement in this neighborhood and output to meet the final conditions. The results obtained can be used for the quasi-optimal solutions and as effective initial approximations for the numerical solution of the trajectory optimization problems described by models of a higher order.

Keywords*: brachistochrone, singular control, phase portrait, thrust control*

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