Mathematical and computer simulation of manipulators with nonlinear geometric constraint

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The article describes mathematical models and solution of stabilization problems of stationary motions for two manipulators with excessive coordinate and nonlinear geometric constraint in the electric drive: a rotating manipulator and a four-wheel mobile manipulator with elastic suspension. The method used here was developed earlier for holonomic and nonholonomic systems with differential constraints using the theory of critical (special) Lyapunov cases in the nonlinear stability theory. The dynamics of the mechanical part of the manipulators is described using equations in the form developed by M.F. Shulgin for systems with redundant coordinates that do not contain joining factor. The voltage at the armature winding of the actuating electric motor is used for control. The second Kirchhoff's law describes the dynamics equation of the motor. The closed system is a system of indirect control. The control law is determined by solution of the linear-quadratic stabilization problem by the Krasovsky method for an isolated subsystem that does not include the critical variable corresponding to the zero root. Coefficients of controlling actions were found by solving the matrix algebraic Riccati equation using programs developed in the MATLAB system and taking into account the conditions imposed by the geometric constraint on the coordinate perturbations.

Keywords: *geometric constraints, redundant coordinates, Shulgin equations, stabilization, stationary motion, manipulator*

REFERENCES

[1] Zenkevich S.L., Yuschenko A.S. *Osnovy upravleniya manipulyatsionnymi robotami* [Principles of robotic manipulator control]. Moscow, BMSTU Publ., 2004, 480 p.

[2] Vukobratovic M., Stokic D., Kircanski N. *Non-Adaptive and Adaptive Control of Manipulation Robots*. Heidelberg, Berlin, Springer-Verlag Publ., 1985 [In Russ.: Vukobratovic M., Stokic D., Kircanski N.Neadaptivnoe i adaptivnoe upravlenie manipulyatsionnymi robotami. Moscow, Mir Publ., 1989, 376 p.].

[3] Matukhin V.I. *Upravlenie mekhanicheskimi sistemami* [Control of mechanical systems]. Moscow, Fizmatlit Publ., 2009, 320 p.

[4] Lurye A.I. *Analiticheskaya mekhanika* [Analytical mechanics]. Moscow, Fizmatlit Publ., 1961, 824 p.

[5] Routh E.J. *Dynamics of a system of rigid bodies*. London, MacMillan and Co. Publ., 1891 [In Russ.: Routh E.J. Dinamika sistemy tverdykh tel]. Moscow, Nauka Publ., 1983, vol. 2, 544 p.].

[6] Min-Sung Koo, Ho-Lim Choi, Jong-Tae Lim. *International Journal of Innovative Computing, Information and Control*, 2012, vol. 8, no. 9, pp. 5999–6009.

[7] Krasinsky A.Ya., Krasinskaya E.M. O dopustimosti linearizatsii uravneniy geometricheskikh svyazey v zadachakh ustoychivosti i stabilizatsii ravnovesiy [On

the admissibility of the linearization of the equations of geometric constraints in problems of stability and stabilization of equilibria]. In: *Sbornik nauchno-metodicheskikh statey. Teoreticheskaya mekhanika. Vyp. 29* [Collection of scientific and methodical articles. Theory of Mechanics. Issue 29]. Samsonov V.A., ed. Moscow, MGU Publ., 2015, pp. 54–65.

[8] Shulgin M.F. O nekotorykh differentsialnykh uravneniyakh analiticheskoy dinamiki i ikh integrirovanii [On some differential equations of analytic dynamics and their integration]. In: *Trudy. Novaya seriya. Vyp. 144: Fiziko-matematicheskie nauki. Kn. 18* [Proceedings. New series. Issue 144: Physics and Mathematics. Book 18]. Tashkent, SAGU Publ., 1958, 183 p.

[9] Neimark Yu.I., Fufayev N.A. *Dinamika negolonomnykh system* [Dynamics of non-holonomic systems]. Moscow, Nauka Publ., 1967, 519 p.

[10] Krasinskaya E.M., Krasinsky A.Ya., Obnosov K.B. O razvitii nauchnykh metodov shkoly M.F. Shulgina v primenenii k zadacham ustoychivosti i stabilizatsii mekhatronnykh system s izbytochnymi koordinatami [On the development of scientific methods of the M.F. Shulgin's school applied to problems of stability and stabilization of equilibria of mechatronic systems with redundant coordinates]. In: *Sbornik nauchno-metodicheskikh statey. Teoreticheskaya mekhanika. Vyp. 28* [Collection of scientific and methodical articles. Theory of mechanics. Issue 28]. Samsonov V.A., ed. Moscow, MGU Publ., 2012, pp. 169−184.

[11] Krasinskaya E.M., Krasinsky A.Ya. *Nauka i obrazovanie: electronnyy nauchno-tekhnicheskiy zhurnal — Science and Education: Electronic Scientific and technical Journal*, 2013, no. 3. DOI: 10.7463/0313.0541146

[12] Lyapunov A.M. *Sobranie sochineniy. V 6 tomakh* [Collected works. In 6 volumes]. Moscow, Leningrad, USSR Academy of Sciences Publ., 1956, vol. 2, 472 p.

[13] Malkin I.G. *Teoriya ustoychivosti dvizheniya* [Theory of motion stability]. Moscow, Nauka Publ., 1966, 530 p.

[14] Krasinskaya-Tumeneva E.M., Krasinsky A.Ya. O vliyanii struktury sil na ustoychivost polozheniy ravnovesiya negolonomnykh system [On the influence of the structure of forces on the stability of nonholonomic system equilibrium positions]. In: *Sbornik trudov "Voprosy vychislitelnoy i prikladnoy matematiki"* [Problems of computational and applied mathematics. Collection of works]. Tashkent, 1977, no. 45, pp. 172–186.

[15] Krasinskaya E.M. *Prikladnaya matematika i mekhanika — Journal of Applied Mathematics and Mechanics*, 1983, vol. 47, no. 2, pp. 302–309.

[16] Krasinsky A.Ya. *Prikladnaya matematika i mekhanika — Journal of Applied Mathematics and Mechanics*, 1988, vol. 52, no. 2, pp. 194–202.

[17] Krasinsky A.Ya. *Prikladnaya matematika i mekhanika — Journal of Applied Mathematics and Mechanics*, 1992, vol. 56, no. 6, pp. 939–950.

[18] Krasinsky A.Ya. *Avtomatika i telemekhanika — Automation and Remote Control*, 2004, no. 1, pp. 97–103.

[19] Krasinskaya E.M., Krasinsky A.Ya. Ob odnom metode issledovaniya ustoychivosti i stabilizatsii ustanovivshikhsya dvizheniy mekhanicheskikh system s izbytochnymi koordinatami [On a method of researching the stability and stabilization of steady motions of mechanical systems with redundant coordinates]. *Trudy XII Vserossiyskogo soveschaniya po problemam upravleniya. Moskva, 16–19 iyunya 2014 g.* [Proceedings of the XII National conference on problems of control. Moscow, June *16–19, 2014*]. Moscow, Institut problem upravleniya RAN Publ., 2014, pp. 1766–1778.

[20] Krasinskiy A.Ya., Krasinkaya E.M. *Avtomatika i telemekhanika — Automation and Remote Control*, 2016, no. 8, pp. 85–100.

[21] Krasinskiy A.Ya., Krasinkaya E.M., Ilyina A.N. *International Robotics and Automation Journal*, 2017, vol. 2, no. 1. DOI: 10.15406/iratj.2017.02.00007

[22] Aizerman M.A., Gantmacher F.R. *Zeitschrift für Angewandte Mathematik und Mechanik*, 1957, Bd. 37, No. 1/2, S. 74–75. DOI: 10.1002/zamm.19570370112

[23] Rumyantsev V.V. *Ob ustoychivosti statsionarnykh dvizheniy sputnikov* [On the stability of satellite stationary motions]. Moscow, VTs AN USSR Publ., 1967, 155 p.

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