N.E. Zhukovsky's strain ellipsoid taking into account terms of the second order of smallness

© V.M. Ovsyannikov

Moscow State Academy of Water Transport — Branch of Admiral Makarov State University of Maritime and Inland Shipping, Moscow, 117105, Russia
Noyabrsk Institute of Oil and Gas (branch) of Industrial University of Tyumen, Noyabrsk, 629802, Yamalo-Nenets Autonomous Okrug, Tyumen Region, Russia

The study deals with the physical interpretation of the terms of the second order of smallness found in the continuity equation. The article shows that these terms, which are usually discarded, also contribute to, for instance, periodic wave generation, increasing the vibration intensity computed by L.D. Landau and E.M. Lifshitz more than twofold. N.E. Zhukovsky disregarded the terms of the second order of smallness with respect to strain time or flow time when writing down the continuity equation for plotting the strain ellipsoid. He did calculate a number of additional terms; this is why he could have balanced the amount of substance taking into account terms of the second and third orders of smallness. We also identified that the expression for the curl of the velocity vector in terms of angular velocity is inaccurate: at the level of accuracy defined by terms of the second order of smallness there should be additional terms in the velocity curl as a function of angular velocity. We analysed the continuity equation found in one of the articles by L. Euler that he presented in 1752 at the Royal Prussian Academy of Sciences, and we show that for incompressible fluid those additional terms create local nonconservation. This case should only be considered as a model one that is not possible in reality. For compressible gas this local nonconservation becomes periodic and describes actually existing flows with periodic pressure waves, or sound, generated by the flow. We determined that in equations of compressible gas dynamics these terms of the second order of smallness with respect to motion time found in the non-homogeneous part of the wave equation lead to generation of sound and self-excited vibrations that are not connected to external factors affecting the flow.

Keywords: *continuity equation, terms of higher orders of smallness with respect to time, self-excited vibrations, wave equation, strain ellipsoid*

REFERENCES

[1] Zhukovsky N.E. *Kinematika zhidkogo tela*. *Diss. mag. prikl. matem.* [Kinematics of the fluid body. Master of applied math. diss.]. Moscow, University Press, 1876, 155 p.

[2] Bubnov V.A. Fizicheskie printsipy gidrodinamicheskikh dvizheniy [Physical principles of hydrodynamic motion]. *Problemy aksiomatiki v gidrogazodinamike* [Axiomatics problems in fluid and gas dynamics], 1997, no. 4, pp. 206–269.

[3] Bubnov V.A. Kinematika zhidkoy chastitsy [Kinematics of a fluid particle]. *Problemy aksiomatiki v gidrogazodinamike* [Axiomatics problems in fluid and gas dynamics], 1999, no. 7, pp. 11–29.

[4] Kochin N.E., Kibel I.A., Roze N.V. *Teoreticheskaya gidromekhanika* [Theoretical fluid mechanics]. Part II. Moscow, State Publishing House of Technical and Theoretical Literature, 1941, vol. 1, 348 p.

[5] Euler L. Principes generaux du movement des fluids [General principles of fluid motion]. *Opera omnia*, Ser. II, vol. 12, pp. 54–91.

[6] Zhukovsky N.E. *Polnoe sobranie sochineniy* [Collected works]. *Vol. 2: Gidrodinamika* [Fluid dynamics]. Moscow, Leningrad, United Science and Engineering Publishing House of People's Commissariat of Heavy Industry, 1935.

[7] Euler L. *Commentationes Mechanicae ad Theoriam Corporum Pertinentes* [Studies in mechanics: on the theory of relevant bodies]. Vol. 1. C.A. Truesdell, ed. Lausannae, 1954.

[8] Ovsyannikov V.M. *Lokalnoe differentsialnoe nesokhranenie pri integralnom sokhranenii v gazovoy dinamike* [Local differential nonconservation combined with integral conservation in gas dynamics]. Moscow, Sputnik Plyus Publ., 2017, 273 p.

[9] Euler L. Principia motus fluidorum [Principles of Fluid Motion]. *Opera omnia*, Ser. II, vol. 12, pp. 133–168. [In Bulg.: Euler L. Printsipi na dvizhenieto na technosti. *Problemy aksiomatiki v gidrogazodinamike* [Axiomatics problems in fluid and gas dynamics], 2017, no. 31, pp. 19–24].

[10] Ovsyannikov V.M. *Volnoobrazovanie i konechnoraznostnoe uravnenie nerazryvnosti Leonarda Eylera* [Wave generation and Leonhard Euler's finite-difference continuity equation]. Moscow, Sputnik Plyus Publ., 2017, 487 p.

[11] Ovsyannikov V.M. Istoriya vyvoda uravneniya nerazryvnosti [History of deriving the continuity equation]. *Sbornik dokladov XI Vserossiyskogo sezda po fundamentalnym problemam teoreticheskoy i prikladnoy mekhaniki. Kazan, 20–24 avgusta 2015* [Proc. of the 11th All-Russian congress on basic problems of theoretical and applied mechanics. Kazan, Aug. 20–24, 2015]. Kazan, 2015, pp. 2823–2824.

[12] Lighthill M.J. *Proceedings of the Royal Society*, A211, 1952; A222, 1954.

[13] Ovsyannikov V.M. Konechno-raznostnoe uravnenie nerazryvnosti Leonarda Eylera [Leonhard Euler's finite-difference continuity equation]. *Problemy aksiomatiki v gidrogazodinamike* [Axiomatics problems in fluid and gas dynamics], 2010, no. 20, 119 p.

[14] Ovsyannikov V.M. Parnost deformatsiy — prichina vozniknoveniya chlenov vysokogo poryadka malosti po vremeni v vyvode Eylera uravneniya nerazryvnosti [Strain coupling as the reason behind terms of higher orders of magnitude with respect to time emerging in Euler's derivation of the continuity equation]. *Tezisy dokladov Sedmykh Polyakhovskikh chteniy Mezhdunarodnoy nauchnoy konferentsii po mekhanike. Sankt-Peterburg, 2–4 fevralya 2015 g.* [Proc. of the 7th Polyakhov readings of International scientific conference in mechanics. Saint Petersburg, February 2–4, 2015]. Saint Petersburg, 2015, p. 135.

[15] Fikhtengolts G.M. *Kurs differentsialnogo i integralnogo ischisleniya* [Course of differential and integral calculus]. Vol. 3. Moscow, State Publishing House for Physical and Mathematical Literature, 1960, 656 p.

[16] Ovsyannikov V.M. *Computational Mathematics and Mathematical Physics*, 2017, vol. 57, no. 5, pp. 876—880. DOI: 10.1134/S0965542517050098

[17] Ovsyannikov V.M. Vvedenie v aksiomaticheskuyu mekhaniku zhidkosti, osnovannuyu na bazisnykh eksperimentakh s zhidkostyu [Introduction to axiomatic fluid mechanics based on reference fluid experiments]. *Problemy aksiomatiki v gidrogazodinamike* [Axiomatics problems in fluid and gas dynamics], 2006, no. 15, pp. 19–51.

Ovsyannikov V.M. graduated from Moscow Institute of Physics and Technology and completed postgraduate studies there. Dr. Sc. (Eng.), Professor, Moscow State Academy of Water Transport — Branch of Admiral Makarov State University of Maritime and Inland Shipping, Noyabrsk Institute of Oil and Gas (branch) of Industrial University of Tyumen. Specialises in radiation gas dynamics, two-phase mixture flows and self-excited vibrations. e-mail: OvsyannikovVM@yandex.ru