Analytical model of the Earth — Moon system gravitational potential in the form of a general solution to the Newtonian three-body problem

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The article presents a mathematical model of the gravitational potential of the Earth — Moon system, which allows analytical calculating the optimal space vehicle trajectories in this celestial mechanical system. This model represents the canonical quaternion generalization of the classical mathematical Aksenov — Grebenikov — Demin model of the Earth gravitational potential. It is shown that the proposed model also realizes the complete separation of the variables in the Hamiltonian of the classical Newtonian three-body problem and corresponds to its analytic solvability, which is the unexpected fact from the classical point of view. Thus, the resulting formulas for an exact general solution of the three-body problem simulate equipotential surfaces of the gravitational potential of the Earth — Moon system. Equipotential lines, in particular, simulate the orbits of space vehicles in the Earth — Moon system. Optimum control of space vehicles is mathematically simulated by a corresponding change in the parameters of the general solution to the Newtonian three-body problem. It is shown that such parametric deformations geometrically correspond to the isometries of the analytic complex three-dimensional Lobachevsky space.

Keywords: *gravitational potential, Earth—Moon system, optimal satellite orbits, Newtonian three-body problem, exact analytic solvability, L-functions of elliptic curves*

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