Numerical and analytical plotting of periodic motion and investigating motion stability in the case of a symmetric satellite

© E.A. Sukhov[1], B.S. Bardin[1,2]

[1] Moscow Aviation Institute (MAI), Moscow, 125993, Russia
[2] Blagonravov Mechanical Engineering Research Institute of the Russian Academy of Sciences, Moscow, 101990, Russia

A specific case of motion of a solid dynamically symmetric satellite along a circular orbit in reference to the centre of mass is its hyperboloid precession. If the hyperboloid precession is stable, the equations of satellite motion allow for existence of periodic motion families that describe the oscillations of the satellite's dynamical symmetry axis in the vicinity of the hyperboloid precession. It is possible to derive these families in the form of convergent series in powers of the small parameter, i.e. the oscillation amplitude. There exist two types of these motions: short-term and long-term. If the amplitude is not small, it is necessary to employ a numerical method in order to plot the motions. In the three-dimensional space of the problem parameters, the authors plotted the region where long-term motions exist that stem from the hyperboloid precession of a symmetric satellite. We deal with the cases of resonance being present and third order resonance being absent. We conducted a first-order investigation of the orbital stability problem for long-term motions. We provide the problem statement and the results of plotting the periodic motions analytically in the absence of resonances. We describe in brief the method for plotting the periodic solution families numerically. We present the results of numerical and analytical plotting of long-term solution families stemming from the hyperboloid precession in the vicinity of the resonance. We draw conclusions on the first-order orbital stability of said solutions for small amplitudes.

Keywords: *Hamiltonian system, periodic motions, dynamically symmetric satellite, numerical continuation of solution families, regular precession, orbital stability, resonance*

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Sukhov E.A. (b. 1990) graduated from Moscow Aviation Institute in 2014. Post-graduate student, Department of Mechatronics and Theoretical Mechanics, Moscow Aviation Institute; Research Assistant, Science and Research Department, Moscow Aviation Institute. Specialises in stability theory, non-linear oscillation theory, celestial mechanics, satellite dynamics. e-mail: sukhov.george@gmail.com

Bardin B.S. (b. 1966) graduated from Moscow Aviation Institute in 1989. Dr. Sc. (Phys.-Math.), Head of the Department of Mechatronics and Theoretical Mechanics, Moscow Aviation Institute; Professor, Russian Academy of Sciences. Specialises in stability theory, non-linear oscillation theory, celestial mechanics, satellite dynamics.
e-mail: bsbardin@yandex.ru