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# Comparative stress analysis in nonsymmetrical multilayer composite plates in the asymptotic theory and three-dimensional finite element calculation

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The article analyzes the accuracy of previously developed multilayer thin plates' asymptotic theory. We compare the solution results of the bending problem for a multilayer asymmetric plate under pressure obtained in the asymptotic theory and in the exact three-dimensional elasticity theory. The problem solution in the asymptotic theory for the asymmetric plate case was obtained for the first time. The paper shows that the plate layers' arrangement asymmetry leads to the longitudinal plate displacements at transverse pressure. We used a software finite-element ANSYS package with a specially constructed finite element mesh to solve the elasticity theory three-dimensional problem. This grid allows for a finite element nodes thickening along the plate thickness while maintaining a relatively small grid lateral elements total number. The paper compares all stresses distributed along the plate thickness, obtained by means of the asymptotic theory and finite elements method. We show that the developed asymptotic theory provides high solution accuracy for all stress components, including transverse and shear stresses.

**Keywords:** asymptotic theory of plates, multilayer thin plates, asymmetric plates, finite element method, lateral stresses, numerical simulation

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