Spacecraft discrete orientations

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We consider the problem of modeling the variety of spacecraft discrete orientations which can be used in testing the systems of controlling the spacecraft positions in space. The criterion of equable filling the orientational space forms the basis of this model. We use the proprietary universal methodology for the random distribution of the points on the smooth regular surfaces in the three-dimensional Euclidean space and its generalization for the hypersurfaces defined by the parameter mode in multidimensional spaces. We have identified the function of the orientational parameters joint distribution density in the form of Euler angles with the uniform distribution of the points on the surface in the three-dimensional space. It is established that the uniformly distributed points on the surface of the three-dimensional unit hypersphere in the four-dimensional Euclidean space define the corresponding Rodriguez—Hamilton parameters set, that confirms the fact of two-sheeted covering the special orthogonal $SO(3)$ matrixes group by the three-dimensional hypersphere. We have carried out the transition from the continuous discrete distribution to the uniform one. The article introduces an algorithm for discrete filling the orientational space based on the application of regular centrosymmetrical polyhedrons in the four-dimensional space. The vertices of these polyhedrons form the sets of needed Rodriguez-Hamilton parameters or quaternions. We provide a constructive proof of the formulated algorithm correctness and its illustrating by means of the body position visualization in the three-dimensional space exemplified by creating 12 discrete orientations uniformly filling the orientational space on the basis of the 24-cell in the four-dimensional space. It is shown that in the general case when creating a spacecraft discrete orientations system we can use the information on the vertices coordinates of five regular four-dimensional polyhedrons (hypercube, 16-cell, 24-cell, 120-cell, 600-cell). The article describes the potential area of practical applications for the results obtained.

Keywords: *Rodriguez—Hamilton parameters, quaternions, orientational space, three-dimensional hypersphere, regular polyhedrons, equable filling*

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