On the causes of divergence of calculation results and experiment when determining stability boundaries for the inverted pendulums (by the article of D.J. Acheson and T. Mullin in *Nature*)

© V.A. Gribkov, Ya.D. Gordin

Bauman Moscow State Technical University, Moscow, Russia

The article analyses the results published by D.J. Acheson, T. Mullin in the journal "Nature". The famous article covered the experimental and calculated determination of the stability boundaries for single, double and triple inverted pendulums vibration stabilized. It marked a radical discrepancy between the results of calculations and experiments to double and triple pendulums (lack of coordination between the calculated and experimental boundaries of the stability boundaries). The aim of the paper is to find the causes that led to a significant difference in the position of the calculated and experimental stability boundaries of double and triple inverted pendulum, as well as checking the operability and effectiveness of D.J. Acheson pendulum theorem.

Checking calculation of the boundaries of sustainability was not possible due to lack of some parameters required for solving the problem of pendulum systems. Through the parameters given in the article, using the range of sizes in the core tubular elements pendulums, with the help of numerical experiments in SOLIDWORKS we restored the missing dimensions and inertial characteristics. D.J. Acheson pendulum theorem, using natural frequencies of direct pendulums we received updated stability region. For double and triple pendulums we specified range of the stability region closer to the experimental in a significant range of the excitation parameters. Thus we confirmed performance of D.J. Acheson pendulum theorem. It is proved that the radical divergence between the calculated and experimental boundaries of sustainability in the article by D.J. Acheson and T. Mullin caused a large error in determining the higher natural vibration frequencies of double and triple pendulums (all own frequencies were determined experimentally using the main parametric resonance of pendulums).

Keywords: *inverted physical pendulum, N-linked pendulum, parametric excitation, dynamic stability, experiment*

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Gribkov V.A., Cand Sc. (Eng), associate professor of the Department “Aerospace systems”, Bauman Moscow State Technical University. Research interests — the dynamics and stability of motion of mechanical systems. e-mail: zenit-ab@mail.ru

Gordin Ya.D., a student of the Department “Aerospace systems”, Bauman Moscow State Technical University. Grant holder of the President of the Russian Federation. Research interests — the dynamics of the flight of aircraft, the dynamics of aircraft constructions.
e-mail: yaroslav.gordin@gmail.com