Explicit formulas for the calculation of the complete tensor of the stresses in the monoclinic thin composite shells based on the asymptotic homogenization method

© Yu.I. Dimitrienko, E.A. Gubareva, Yu.V. Yurin

Bauman Moscow State Technical University, Moscow, 105005, Russia

The article presents the results of further development of the previously proposed by the authors' new asymptotic theory of thin multilayer anisotropic shells. The theory is constructed on the equations of the general three-dimensional theory of elasticity by introducing small geometric parameter asymptotic expansions without any hypotheses on stresses and displacements distribution over the thickness. The case of monoclinic layers having at the most 13 independent elastic constants is considered. An algorithm for obtaining explicit analytic formulas for the calculation of the complete stress tensor component distribution over the shell is proposed. The algorithm is based on solving specific local problems of the first, second and third approximations. It allows obtaining expressions for all six components of the stress tensor in a compact closed form, as a function of strain, curvature of the middle surface of the shell, as well as their derivatives with respect to the longitudinal coordinates. These formulas allow calculating all stress tensor components in the shell without additional tasks, using only the solutions of the averaged problem of shell theory.

Keywords: *stress tensor, multilayer thin monoclinic shells, composites, the method of asymptotic averaging, asymptotic theory of shells*

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Dimitrienko Yu. I. (b. 1962) graduated from Lomonosov Moscow State University in 1984. Dr. Sci. (Phys. & Math.), Professor, Head of the Department of Computational Mathematics and Mathematical Physics, Director of Scientific-educational Center of Supercomputer Engineering Modeling and Program Software Development, Bauman Moscow State Technical University. Member of the Russian Academy of Engineering Science. Author of over 350 publications in the field of computational mechanics, gasdynamics, thermomechanics of composite materials, mathematical simulations in material science. e-mail: dimit.bmstu@gmail.com

Gubareva E.A. (b. 1982) graduated from Lomonosov Moscow State University in 2004. Cand. Sci. (Phys.&Math.), Associate Professor, Department of Computational Mathematics and Mathematical Physics, Bauman Moscow State Technical University. Author of 50 scientific publications in the field of composite mechanics, asymptotic analysis, contact mechanics. e-mail: eagubareva@mail.ru

Yurin Yu.V. (b. 1989) graduated from Bauman Moscow State Technical University in 2012. Postgraduate student (Ph.D.), Department of Computational Mathematics and Mathematical Physics, Bauman Moscow State Technical University. Author of 20 publications in the field of composite mechanics. e-mail: yvyurin@yandex.ru