Filtration model through a homogeneous porous medium

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The study considers a model of vertical water transfer in soil; describes the water transfer process by one-dimensional nonlinear parabolic equation. The diffusion coefficient and the soil hydraulic conductivity included in the equation is calculated by van Genuchten formulas widely used in practice. An important model component is the evaporation from the soil surface. The study formulates the problem of determining evaporation as an optimal problem — the one, in which the phase variables are the soil moisture values at different depths, and control is the desired evaporation. The mean-square soil moisture values deviation from some prescribed values derived from calculations based on the hydrological models is the objective function. We solve the numerical optimization by the steepest descent method; the objective function gradient is calculated using the fast automatic differentiation method (FAD).

Keywords: *the steepest descent method, the objective function, the optimal control problem, a method of fast automatic differentiation.*

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