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# Some aspects of solving the optimal control problem on the basis of the maximum principle for non-coplanar interorbital transfer

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*The article considers the approach to solve the problem of determining the optimal thrust vector control of propulsion system according to the criterion of maximization of the payload mass in the orbital unit launch into an arbitrary target orbit, which is non-coplanar to the original one. The problem is solved using Pontryagin's maximum principle, which reduces the problem of optimal control search to solve a boundary value problem. The successful solution of the problem depends on the quality of the initial approximation of its unknown parameters. In solving the problem the parameter continuation method was applied. The technique of operational conversion of the conjugate variable initial values using the symmetry properties of the target orbit inclination with respect to the plane of the initial reference orbit was also applied. As a part of the work numerical modeling of the orbital block motion in the central gravitational field of the Earth with the optimal control, obtained on the basis of the maximum principle, was carried out. The dependence of the optimal launch scenario on the form (eccentricity) of the target orbit was analyzed. The behavior of conjugate variables, determining the vector of optimal control, which delivers a maximum output payload mass with set limits on the duration of transfer, was analyzed for a wide range of target orbit's inclination. The analysis revealed the symmetry properties of the behavior of conjugate variables, the use of which improves the efficiency of finding the optimal solution. The obtained results can be applied in the field of ballistic design calculations related to the development of launch vehicles (boosters, space tugs) and spacecraft for various purposes. They can also be applied for assessing the capacity of existing launch vehicles and determining the first approximation in the development of transfer schemes.*

**Keywords:** Pontryagin's maximum principle, non-coplanar interorbital transfer, optimal control, launch.

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