Theory of plates stability, based on asymptotic analysis of stability theory equations for three-dimensional elastic bodies

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The objective of this research is to develop a theory of elastic stability of thin multilayer plates. The theory is based on general equations of three-dimensional theory of elastic stability by means of introducing the asymptotic expansion over a small parameter, which represents a thickness to length of plate ratio, without any hypothesis about displacements and stress distributions. Within the research, we stated local problems of stability, as well as the averaged equations of plate equilibrium for the ground states and the varied states of the plate. Consequently, we obtained the analytical solution of the local problems, which helped deduce relations for all six components of the stress tensor, including throw-thickness normal stresses and shear stresses for the ground and varied states. Moreover, we found that the averaged equations of plates' stability differ from the classic equations of Kirchoff—Love and Timoshenko's plate theory of stability. It is determined that for orthotropic plates the constitutive relations simplify and become similar to classical relations of thin plates. However, the membrane and flexural stiffness of plates depends on stresses of the ground state. The study is illustrated with an example of calculating a thin orthotropic plate under uniaxial compression. As a result, we obtained an expression for the critical buckling force, which differs from the classical Euler formula in expression for flexural stiffness, which depends on the parameters of the ground state of the plate. The findings of the research show that the difference of the critical force values is the most significant for the plates with strong anisotropic layers.

Keywords*: theory of plates' stability, three-dimensional stability theory, thin multi-layer plates, orthotropic plates, asymptotic expansion.*

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