Elastic beams of minimum weight in the presence of several types of bending loads

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The article considers the problem of optimizing the loaded beam thickness, i.e. minimizing weight of the structure, for given boundary conditions and restrictions of strain capacity. It was found that the mathematical model in this case is the boundary value problem for ordinary differential equation of 4th order. Solving the optimization problem is based on two different approaches. The first one is the classical variational method based on studying the variation of the minimized functional and analyzing the stationary point of the functional. In the second method, the Pontryagin maximum principle is applied to the problem with fixed left and right ends.
Numerical experiments carried out for different types of bending loads, are illustrated by graphs. Comparison of the results shows the equivalence of the two approaches. This significantly extends the range of optimization problems, for solution of which software with models of complex systems is developed.

Keywords: *beam thickness optimization, variational method, principle of maximum.*

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