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# Nonlinear model of blow with dry friction

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*According to the Hertz model of absolutely elastic impact, the contact force of interaction of the bodies depends on deformation as well as it is in statics. K.H. Hunt and F.R.E. Crossley suggested that at collision there arises not only an elastic force, but also viscous friction between particles of the colliding bodies. Thus the restoration coefficient monotonously decreases with increasing impact velocity. A nonlinear elastic-plastic model of collinear impact of a body about a fixed obstacle is based on the Hertz and Hunt — Crossley models of impact, in which it is assumed that the friction between the particles of colliding bodies is not viscous, but dry. We obtained the first integrals of the equations of the movement in phases of deformation and restoration. The coefficient of restoration and its dependence on a friction constant was defined. We obtained a solution of equations of motion of the body in quadrature; presented the results of mathematical modeling. In the paper we show that absolutely inelastic impact is possible in this model, while elastic collision restitution coefficient does not depend on the speed of the collision.*

**Keywords:** collinear impact, the coefficient of restitution, nonlinear dynamics.

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