Energy spectrum in a problem of a quantum rotator

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The study tested the spectrum of eigenvalues of quantum systems, in the classical limit admitting the existence of quadratic ones in the momenta of the first integrals. For example, we consider the problem of two-dimensional potential well of finite depth and quantum rotator. We made a comparison of bifurcations in classical and quantum problems. The research showed that the presence of the additional integral imposes a partial ban on the existence of separatrix contours. We examined an algebraic structure of classical integrals, which determines the possibility of bringing Hamiltonian function to Liouville type and separation of variables in Hamilton — Jacobi equation, which entails the separation of variables in Schrödinger equation.

Keywords: *dynamical systems, bifurcation, Hamiltonian systems.*

REFERENCES

[1] Eleonskiy V.M., Kulagin N.E. Domennye granitsy, solitony i fazovye perekhody [Domain walls, solitons, and phase transitions]. *Sb. Issledovaniya po fizike kineticheskikh yavleniy* [Collected studies in physics of kinetic phenomena]. Sverdlovsk, Ufa Sci. Cen. Acad. Sci. USSR, 1984, pp. 56–65.

[2] Eleonskiy V.M., Kulagin N.E. *ZhETF — Journal of Experimental and Theoretical Physics,* 1983, vol. 84, iss. 2, pp. 616–628.

[3] Eleonskiy V.M., Kulagin N.E. *ZhETF — Journal of Experimental and Theoretical Physics,* 1987, vol. 93, iss. 4 (10), pp. 1436–1446.

[4] Prudnikov A.P., Brychkov Yu.A, Marychev O.I. *Integraly i ryady. Elementarnye funktsii* [Integrals and series. Elementary functions]. Moscow, Nauka Publ., 1981, 800 p.

[5] Abromovits M., Stigan I., eds. *Spravochnik po spetsial'nym funktsiyam* [Handbook of Mathematical Functions]. Moscow, Nauka Publ., 1979, 832 p.

[6] Beitmen G., Erdeii A. *Vysshie transtsendentnye funktsii. Ellipticheskie i avtomorfnye funktsii. Funktsii Lame i Mat'e* [Higher transcendental functions. Elliptic and automorphic functions. Lame and Mathieu functions]. Moscow, Nauka Publ., 1967, 300 p.

[7] Yanke E., Emde F., Lesh F. *Spetsial'nye funktsii. Formuly, grafiki, tablitsy* [Special functions. Formulas, graphs, tables]. Moscow, Nauka Publ., 1977, 344 p.

[8] Gurchenkov A.A., Esenkov A.S., Tsurkov V.I. *Izvestiya RAN. Teoriya i sistemy upravleniya — Proceedings of the Russian Academy of Sciences. Control Theory and Systems,* 2006, no. 3, pp. 82–89.

[9] Gurchenkov A.A., Esenkov A.S., Tsurkov V.I. *Izvestiya RAN. Teoriya i sistemy upravleniya — Proceedings of the Russian Academy of Sciences. Control Theory and Systems,* 2006, no. 1, pp. 141–148.

[10] Gurchenkov A.A., Esenkov A.S., Tsurkov V.I. *Avtomatika i telemekhanika — Automatics and Telemechanics,* 2007, no. 2, pp. 81–94.

[11] Gurchenkov A.A., Nosov M.V, Tsurkov V.I. *Control of Fluid-Containing Rotating Rigid Bodies.* CRC Press, 2013, 147 p.

[12] Mironov A.A., Tsurkov V.I. *Zhurnal vychislitel'noi matematiki i matematicheskoi fiziki — Journal of Computational Mathematics and Mathematical Physics,* 1993, vol. 33, no. 2.

[13] Tsurkov V.I. Aggregation in a branch manufacturing problem and its extension. *Proc. 13th IFAC Symposium on Information Control Problems in Manufacturing, INCOM '09,* 2009, pp. 310–312.

[14] Mironov A.A., Tsurkov V.I. *Dokl. Akademii nauk — Acad. Sci. reports,* 1994, vol. 336, no. 1.

[15] Gurchenkov A.A. *Izv. vuzov. Ser. Priborostroenie — University Proced. Ser. Instrument Engineering,* 2001, vol. 44, no. 2, p. 44.

[16] Gurchenkov A.A. *Inzhenerno-fizicheskiy zhurnal — Journal of Engineering Physics,* 2002, vol. 75, no. 3, pp. 28–32.

[17] Gurchenkov A.A., Yalamov Y.I. *Doklady Physics,* 2002, vol. 47, no. 1, pp. 25–28.

[18] Gurchenkov A.A. *J. of Engineering Physics and Thermo Physics,* 2002, vol. 75, no. 3, p. 554.

[19] Gurchenkov A.A. *Prikladnaya mekhanika i tekhnicheskaya fizika — Applied Mechanics and Technical Physics,* 2001, vol. 42, no. 4, pp. 48–51.

[20] Gurchenkov A.A., Korneev V.V., Nosov M.V. *Prikladnaya matematika i mekhanika — Applied Mathematics and Mechanics,* 2008, vol. 72, no. 6, pp. 904–911.

[21] Gurchenkov A.A. *Dokl. Akademii nauk — Acad. Sci. reports,* 2002, vol. 382, no. 4, p. 476.

[22] Gurchenkov A.A. *Prikladnaya matematika i mekhanika — Applied Mathematics and Mechanics,* 2002, vol. 66, iss. 2, pp. 251–255.

[23] Gurchenkov A.A., Eleonskiy V.M., Kulagin N.E. *Sloistye struktury v nelineinykh vektornykh polyakh* [Layered structures in nonlinear vector fields]. Moscow, Comp. Center RAS Publ., 2007, 177 p.

[24] Gurchenkov A.A., Kulagin N.E. *Ob uzorakh simmetrii v prostykh modeliakh nelineinogo skaliarnogo polia* [Patterns of symmetry in simple models of nonlinear scalar field]. Moscow, Comp. Center RAS, 2004, 84 p.

[25] Gurchenkov A.A., Moroz I.I., Popov N.N. *Inzhenernyi zhurnal: nauka i innovatsii — Engineering Journal: Science and Innovation,* 2013, iss. 9. Available at: http://engjournal.ru/catalog/appmath/hidden/1166.html

[26] Gurchenkov A.A., Romanenkov A.M. *Inzhenernyi zhurnal: nauka i innovatsii — Engineering Journal: Science and Innovation,* 2013, iss. 2. Available at: http://engjournal.ru/catalog/appmath/hidden/613.html

[27] Gurchenkov A.A., Egorova E.K. *Inzhenernyi zhurnal: nauka i innovatsii — Engineering Journal: Science and Innovation,* 2013, iss. 12. Available at: http:// engjournal.ru/catalog/appmath/hidden/1167.html

[28] Gurchenkov A.A. *Inzhenernyi zhurnal: nauka i innovatsii — Engineering Journal: Science and Innovation,* 2013, iss. 2. Available at: http://engjournal.ru/catalog/appmath/hidden/603.html

[29] Gurchenkov A.A. *Inzhenernyi zhurnal: nauka i innovatsii — Engineering Journal: Science and Innovation,* 2012, iss. 7. Available at: http://engjournal.ru/articles/297/297.pdf

[30] Gurchenkov A.A. *Dinamika zavikhrennoi zhidkosti v polosti vrashchayushchegosya tela* [Dynamics of swirling liquid in the cavity of the rotating body]. Moscow, Fizmatlit Publ., 2010, 221 p.

[31] Gurchenkov A.A., Nosov M.V., Tsurkov V.I. *Upravlenie vrashchayushchimisya tverdymi telami s zhidkostyu* [Control of rotating solids with the fluid]. Moscow, Fizmatlit Publ., 2011, 202 p.

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